DISCOVERY OF EIGHT VARIABLE STELLAR SPECTRA

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In three earlier communications to the PROCEEDINGS various phases of the problem of Cepheid variation have been considered.[1] No further introduction to the present note is necessary beyond the statement that the periodic change in color, which appears to be typical of all Cepheids, has already been found to correspond to normal changes of spectral class for two cluster-type stars, RS Boötis and RR Lyrae, and for one variable of longer period, δ Cephei. In order to test to what extent the inconstancy of spectrum is a general phenomenon of Cepheids, some 150 spectrograms of representative variables of this class have been made with the 10-inch portrait lens and objective prism. The periods of the stars investigated range from 9 hours to 27 days; many are those for which spectroscopic orbits have been computed; some are well-known naked eye variables, others are much fainter stars; for some, changes in color have been suspected from studies of the light curves, for others the maximum intensity of the spectrum has been observed to shift toward the blue upon the approach to maximum light. For none, however, has it been suggested, so far as I know, that the spectrum changes periodically along the normal spectral series.

Following the numerical method of classification recently described by Adams,[2] the change of spectrum of Cepheids is susceptible of easy detection, even when the variation is very small. For example, in the work with low dispersion on F-type spectra the relative intensity of the *G* band and the hydrogen line $H\gamma$ is particularly capable of showing small changes of type. The results can be summarized briefly.

The table contains data relative to the spectra of eleven stars (all that are now known to have variable spectra), and some information concerning their light fluctuations. The observed range of spectrum variation is in nearly every case smaller than the probable total range, as observations at the exact time of maximum and minimum were not made. The accompanying figure contains two objective-prism spectra of δ Cephei, showing the conspicuous change, from minimum to maximum light, in the relative intensity of the spectral lines. The more detailed appearance with high dispersion of a

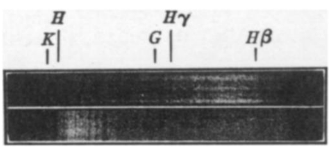

FIG 1. SPECTRA OF δ CEPHEI NEAR MAXIMUM (ABOVE) AND MINIMUM OF LIGHT (enlarged five times).

Eleven Cepheids with Variable Spectra

STAR	MAXIMUM MAGNITUDE	RANGE OF VARIATION	PERIOD	NUMBER OF SPECTROGRAMS	OBSERVED SPECTRUM VARIATION
		mag.	*days*		
TU Cassiopeiae....	7.3	1.1	2.139	9	F0 to F6
SU Cassiopeiae....	5.9	0.4	1.950	19	A8 to F5
SZ Tauri.........	7.2	0.5	3.148	11	F4 to G2
T Monocerotis...	6.0	0.8	27.012	6	F4 to F8
RT Aurigae.......	5.0	0.9	3.728	12	A8 to G0
W Geminorum...	6.4	1.3	7.916	10	F3 to G0
RS Boötis........	8.9	1.1	0.377	13	B8 to F0
X Sagittarii......	4.4	0.6	7.012	5	F2 to G
Y Ophiuchi......	6.2	0.8	17.121	4	F5 to G0
RR Lyrac........	6.8	0.9	0.567	17	B9 to F2
δ Cephei........	3.5	0.8	5.366	21	F2 to G3

smaller section of the spectrum has been shown in an earlier communication by Adams and Shapley.

Every variable for which the present test is sufficient was found to vary in spectrum. It appears safe to infer, therefore, that all Cepheids (including the cluster-type), besides being variable in light and in velocity, vary periodically in spectral class as well.

[1] Shapley and Shapley, these PROCEEDINGS, **1**, 452 (1915); Shapley, *Ibid.*, **2**, 132 (1916) Adams and Shapley, *Ibid.*, **2**, 136 (1916).

[2] These PROCEEDINGS, **2**, 143 (1916).

ON THE LINEAR DEPENDENCE OF FUNCTIONS OF SEVERAL VARIABLES, AND CERTAIN COMPLETELY INTEGRABLE SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

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The study of an ordinary homogeneous linear differential equation of the nth order leads very naturally to the definition of the Wronskian of n solutions of the equation, and thence to the general theory of the linear dependence of n functions of a single variable. This is due to the characteristic property of the said differential equation, viz., that any solution of the equation is linearly dependent upon any fundamental set of solutions. I wish in this note to give some of the results which I have obtained in generalizing the theory of linear dependence to the case of n functions of several independent variables, and also to point out the application of these results to the study of an important